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'Basic Equations of ENGINEERING SCIENCE

SCHAUM'S OUTLINE SERIES

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1.1 CONTINUITY FOR A GENERAL COMPRESSIBLE FLUID

V = Velocity vector
t = Time
ρ = Density

(a) ***Vector.***

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\mathbf{V}) = 0 \qquad 1.1$$

or

$$\frac{D\rho}{Dt} + \rho\nabla\cdot\mathbf{V} = 0 \qquad 1.2$$

where the operator $\frac{D}{Dt}$, called the material, substantial, or Stokes derivative, is given in Section *1.5* or the Appendix.

(b) Cartesian Tensor.

w_i is the velocity in the x_i direction.

$$\frac{\partial\rho}{\partial t} + \frac{\partial(\rho w_i)}{\partial x_i} = 0 \qquad 1.3$$

(c) Cartesian.

u, v, and w are the velocities in the x, y, and z directions respectively.

$$\frac{\partial\rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0 \qquad 1.4$$

(d) Cylindrical.

v_r, v_θ, and v_z are the velocities in the r, θ, and z directions respectively.

$$\frac{\partial\rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(r\rho v_r) + \frac{1}{r}\frac{\partial}{\partial\theta}(\rho v_\theta) + \frac{\partial}{\partial z}(\rho v_z) = 0 \qquad 1.5$$

(e) Spherical.

v_r, v_θ, and v_ϕ are the velocities in the r, 0, and ϕ directions respectively.

$$\frac{\partial\rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\rho v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\rho v_\theta\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}(\rho v_\phi) = 0 \qquad 1.6$$

The orthogonal coordinates are denoted as x_1, x_2, and x_3. The line element of length is given by

$$ds^2 = h_1^2\, dx_1^2 + h_2^2\, dx_2^2 + h_3^2\, dx_3^2 \qquad \text{A.47}$$

and the element of volume as

$$dV = h_1 h_2 h_3\, dx_1\, dx_2\, dx_3 \qquad \text{A.48}$$

The table below lists various orthogonal coordinate systems and the values of the metric coefficients h_1, h_2, and h_3, together with the relationships between the coordinates and the Cartesian coordinates x, y, and z. For a more complete discussion of the general theory of orthogonal transformation theory and the matrix formulation, the reader is referred to the chapter on dynamics (Chapter 4).

Listed below, then, is an outline of the coordinate systems. For a more complete discussion of the physical significance of the coordinate systems the reader is referred to the references at the end of this appendix. In particular, the reference to Margenau and Murphy is useful.

Tables of Orthogonal Coordinate Systems

	Cartesian	Cylindrical $x = r\cos\theta$ $y = r\sin\theta$ $z = z$	Spherical $x = r\sin\theta\cos\phi$ $y = r\sin\theta\sin\phi$ $z = r\cos\theta$	Confocal Ellipsoidal (a, b and c are constants) $x^2 = \frac{(a^2-\lambda)(a^2-\mu)(a^2-\nu)}{(b^2-a^2)(c^2-a^2)}$ $y^2 = \frac{(b^2-\lambda)(b^2-\mu)(b^2-\nu)}{(a^2-b^2)(c^2-b^2)}$ $z^2 = \frac{(c^2-\lambda)(c^2-\mu)(c^2-\nu)}{(a^2-c^2)(b^2-c^2)}$
x_1	x	r	r	λ
h_1^2	1	1	1	$\frac{1}{4}\left\{\frac{(\mu-\lambda)(\nu-\lambda)}{(a^2-\lambda)(b^2-\lambda)(c^2-\lambda)}\right\}$
x_2	y	θ	ϕ	μ
h_2^2	1	r^2	$r^2\sin^2\theta$	$\frac{1}{4}\left\{\frac{(\nu-\mu)(\lambda-\mu)}{(a^2-\mu)(b^2-\mu)(c^2-\mu)}\right\}$
x_3	z	z	θ	ν
h_3^2	1	1	r^2	$\frac{1}{4}\left\{\frac{(\lambda-\nu)(\mu-\nu)}{(a^2-\nu)(b^2-\nu)(c^2-\nu)}\right\}$

	Prolate Spheroidal $x = a\sinh u\sin v\cos\phi$ $y = a\sinh u\sin v\sin\phi$ $z = a\cosh u\cos v$	Oblate Spheroidal $x = a\cosh u\sin v\cos\phi$ $y = a\cosh u\sin v\sin\phi$ $z = a\sinh u\cos v$	Elliptic Cylindrical $x = a\cosh u\cos v$ $y = a\sinh u\sin v$ $z = z$
x_1	u	u	u
h_1^2	$a^2(\sinh^2 u + \sin^2 v)$	$a^2(\sinh^2 u + \cos^2 v)$	$a^2(\sinh^2 u + \sin^2 v)$
x_2	v	v	v
h_2^2	$a^2(\sinh^2 u + \sin^2 v)$	$a^2(\sinh^2 u + \cos^2 v)$	$a^2(\sinh^2 u + \sin^2 v)$
x_3	ϕ	ϕ	z
h_3^2	$a^2(\sinh^2 u\sin^2 v)$	$a^2\cosh^2 u\sin^2 v$	1

Vector Operations in Orthogonal Curvilinear Coordinates.

Gradient:

$$(\nabla\Phi)_1 = \frac{1}{h_1}\frac{\partial\Phi}{\partial x_1} \qquad \text{A.49}$$

$$(\nabla\Phi)_2 = \frac{1}{h_2}\frac{\partial\Phi}{\partial x_2}$$

$$(\nabla\Phi)_3 = \frac{1}{h_3}\frac{\partial\Phi}{\partial x_3}$$

Divergence:

$$\nabla\cdot\mathbf{A} = \frac{1}{h_1h_2h_3}\left[\frac{\partial}{\partial x_1}(h_2h_3A_1) + \frac{\partial}{\partial x_2}(h_3h_1A_2) + \frac{\partial}{\partial x_3}(h_1h_2A_3)\right] \qquad \text{A.50}$$

Curl:

$$(\nabla\times\mathbf{A})_1 = \frac{1}{h_2h_3}\left[\frac{\partial}{\partial x_2}(h_3A_3) - \frac{\partial}{\partial x_3}(h_2A_2)\right] \qquad \text{A.51}$$

$$(\nabla\times\mathbf{A})_2 = \frac{1}{h_1h_3}\left[\frac{\partial}{\partial x_3}(h_1A_1) - \frac{\partial}{\partial x_1}(h_3A_3)\right]$$

$$(\nabla\times\mathbf{A})_3 = \frac{1}{h_1h_2}\left[\frac{\partial}{\partial x_1}(h_2A_2) - \frac{\partial}{\partial x_2}(h_1A_1)\right]$$

Laplacian:

$$\nabla^2\Phi = \frac{1}{h_1h_2h_3}\left[\frac{\partial}{\partial x_1}\left(\frac{h_2h_3}{h_1}\frac{\partial\Phi}{\partial x_1}\right) + \frac{\partial}{\partial x_2}\left(\frac{h_3h_1}{h_2}\frac{\partial\Phi}{\partial x_2}\right) + \frac{\partial}{\partial x_3}\left(\frac{h_1h_2}{h_3}\frac{\partial\Phi}{\partial x_3}\right)\right] \qquad \text{A.52}$$

Material Derivative:

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \frac{w_1}{h_1}\frac{\partial}{\partial x_1} + \frac{w_2}{h_2}\frac{\partial}{\partial x_2} + \frac{w_3}{h_3}\frac{\partial}{\partial x_3} \qquad \text{A.53}$$

where w_1, w_2, and w_3 are the velocities in the coordinate directions.

A.5 TRIGONOMETRIC RELATIONSHIPS

(a) Trigonometric Identities.

$$\sin^2\theta + \cos^2\theta = 1 \qquad \text{A.54}$$

$$1 + \tan^2\theta = \sec^2\theta \qquad \text{A.55}$$

$$1 + \text{ctn}^2\theta = \csc^2\theta \qquad \text{A.56}$$

$$\sin 2\theta = 2\sin\theta\cos\theta \qquad \text{A.57}$$

$$\cos 2\theta = 2\cos^2\theta - 1 = \cos^2\theta - \sin^2\theta \qquad \text{A.58}$$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} \qquad \text{A.59}$$